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Axisymmetric approach of a solid sphere toward a non-deformable planar slip interface in the normal stagnation flow—development of global rational approximations for resistance coefficients

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Abstract

In this paper a model was developed to describe the hydrodynamic force acting on the particle as it approaches a bubble with a mobile surface in an axisymmetric liquid flow. The particle size was considered to be sufficiently small relative to the bubble size that the bubble surface could be locally approximated to a planar interface. The model incorporated a bispherical coordinate system to derive a stream function for the liquid flow disturbed by the particle. The stream function was then used to calculate the hydrodynamic force acting on a particle of radius, R, as a function of the separation distance, H, from the bubble surface. The force equation was related to the modified Stokes equation to obtain an exact numerical solution for the correction factor, f_2. Finally, simplified analytical rational approximations for the whole range of the separation distance are presented, which are in good agreement with the exact numerical result, and can be readily applied to more general mineral flotation applications.

Keywords: Drag force; Stokes correction factor; Particle–flow interaction; Gas–liquid–solid multiphase flow systems

1. Introduction

The hydrodynamic interaction between fine particles and rising bubbles plays an important role in a number of multiphase processes, including the particle separation by froth flotation (Nguyen,

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1999; Ralston et al., 1999; Schulze, 1983). In the process of froth flotation separation, the hydrodynamic interaction between fine solid particles and rising air bubbles strongly influences the collection of particles onto the bubble surface (Nguyen, 1999; Ralston et al., 1999). The bulk hydrodynamics influences the collision (encounter) between bubbles and particles (Schulze, 1989; Yoon and Luttrell, 1989). The short-range hydrodynamic interaction influences the attachment of solid particles to the bubble surface, which occurs when the particle-bubble separation is sufficiently small, quantitatively, within a few particle diameters (Schulze, 1983). This short-range hydrodynamic interaction is known to increase the hydrodynamic resistance on the particle, which may lead to the situation where no attachment of the particle onto the bubble is possible if no strong attractive surface forces exist between the bubble and the particle. In this paper we focus only on the short-range hydrodynamic interaction, taking place between a spherical particle and a bubble in the direction perpendicular to the bubble surface. In this direction the interaction between the hydrodynamic and surface forces is the most significant. We also assume that the particle is significantly small that the local geometry of the particle–bubble surface can be approximated to the particle–planar surface geometry. Furthermore, it can be assumed that the undisturbed-by-the-particle flow of liquid close to the bubble surface is a creeping flow, and, thus, it can be decomposed into two independent additive flows, namely the flows normal and tangential to the bubble surface. The former flow is therefore a stagnation flow while the latter flow is a cross flow. Finally we can physically approximate the short-range hydrodynamic interaction in the perpendicular direction to the short-range hydrodynamic interaction between a solid sphere and a planar surface in a stagnation flow.

When the separation distance asymptotically approaches zero, the framework of the lubrication approximation can be applied to the solution to the short-range hydrodynamic interaction (Russel et al., 1989). This usually yields Eq. (1), which describes the dependence of the drag force, F, on the separation distance, H.

$$F = -6\pi\mu RVf_1, \tag{1}$$

where V is the particle velocity relative to the bubble surface, R is the particle radius and μ is the liquid viscosity. In Eq. (1), function f_1 describes the deviation of the drag force from the Stokes law and is referred to as the correction factor. It describes the dependence of F on the separation distance H and is given by Eq. (2).

$$f_1 = \frac{R}{\gamma H}. \tag{2}$$

Parameter γ in Eq. (2) accounts for the mobility of the bubble surface: $\gamma = 4$ if the bubble surface is mobile and $\gamma = 1$ if the bubble surface is immobile. However, Eq. (2) for the correction factor is correct only in the limit $H \to 0$ of the framework of the lubrication approximation. It does not reduce to the correct asymptotic value of 1 in the limit $H \to \infty$ when the particle is far from the surface. In this case the solution to the full Navier–Stokes equation of the creeping flow confined between a particle and a planar mobile interface gives (Adamczyk et al., 1983; Brenner, 1961)

$$f_1 = \frac{4}{3}\sinh\alpha \sum_{m=1}^{\infty} \frac{m(m+1)}{(2m-1)(2m+3)} \left[\frac{4\cosh^2(m\alpha + \alpha/2) + (2m+1)^2 \sinh^2\alpha}{2\sinh(2m\alpha + \alpha) - (2m+1)\sinh 2\alpha} - 1 \right], \tag{3}$$

where α is a function of the separation distance, H, between the surfaces, and is given by

$$\alpha = \ln\left\{\frac{H}{R} + 1 + \sqrt{\left(\frac{H}{R}\right)^2 + 2\frac{H}{R}}\right\}. \tag{4}$$

The short-range hydrodynamic drag described by Eqs. (1)–(3) is applied to the situation when the undisturbed (far-field) liquid is stationary. If the far-field liquid is not stationary such as in the industrial processes, the short-range hydrodynamic force, F, acting on the sphere can be described by the modified Stokes equation, i.e.

$$F = -6\pi\mu RVf_1 + 6\pi\mu RWf_2, \tag{5}$$

where W is the undisturbed velocity of liquid. f_2 is another correction factor, which describes the deviation from the Stokes law of the drag force due to the undisturbed liquid flow. The first term on the right hand side of Eq. (5) represents the drag force due to the particle motion. The second term is defined as F_2 in this paper

$$F_2 = 6\pi\mu RWf_2. \tag{6}$$

For particles undergoing Stokesian motion in the bulk phase the correction factors, f_1 and f_2 are equal to unity, and Eq. (5) reduces to the well-known Stokes' drag force equation, with $(V - W)$ being the particle slip velocity. However, in the Stokes regime of the liquid flow confined between surfaces the correction factors are no longer equal to unity. The correction factor f_1 is described by Eq. (3). Expressions need to be obtained for the correction factor f_2.

The emphasis on bubbles with a mobile surface given in this paper also requires justification. Recent studies (Dai et al., 1998; Nguyen, 1998; Schulze, 1992) have shown that it is more appropriate for flotation applications to assume a mobile gas–liquid interface for a rising bubble since the surface contamination is likely to be swept to the rear of the bubble due to the liquid motion. This leaves the front region of the bubble relatively free from contaminants where the particles are most likely to attach themselves. The difference between the mobile and immobile bubbles can be quantified (Nguyen, 1999) by the order of magnitude of the liquid flow velocity, i.e.

Mobile bubbles: $W \sim (z - z^2)$ and Immobile bubbles: $W \sim (z^2)$,

where z is the distance measured from the bubble surface and scaled by dividing by the bubble radius. From the above order-of-magnitude analysis, it follows that different expressions for f_1 and f_2 should be used for mobile and immobile bubbles.

The aim of this paper is to derive an expression for the stream function which describes the liquid motion around a particle as it approaches a mobile planar (bubble) surface. The stream function is then used to obtain a simple expression for the hydrodynamic force, F_2 acting on the particle. It should be noted that in the development of the model the second-order prediction of the undisturbed flow velocity, W, is used since the first-order prediction is usually poor for the modelling of the particle–bubble attachment interaction (Dobby and Finch, 1986; Nguyen, 1993).

2. Mathematical formulation of the problem

The physical approximation of the bubble–particle interaction in the direction normal to the bubble surface to the interaction between a particle and a plane free surface is illustrated in Fig. 1. We use a local cylindrical coordinate system (ω, φ, z), as shown in Fig. 1, to describe the hydrodynamic interaction. The origin of the system is the projection of the particle centre on the planar (bubble) surface, and the z-axis passes through the centre of the particle, while the ω and φ axes lies on the planar surface. Since there is rotational symmetry about z-axis, only the z- and ω-coordinates are used in the modelling.

2.1. Stream function of the undisturbed axisymmetric stagnation flow

Firstly, it is required to quantify the axisymmetric stagnation flow of the liquid close to the interface in the absence of the particle. This is given by the normal velocity component, $\vec{W}$, around the bubble (Nguyen, 1999), i.e.

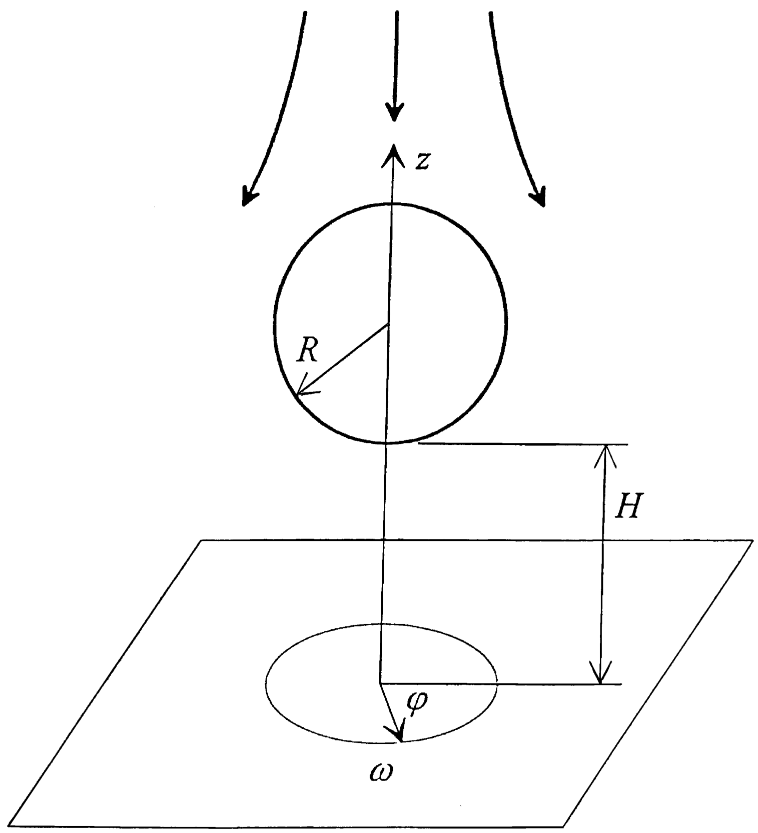

Fig. 1. Spherical solid particle with radius, R, close to a mobile planar (bubble) surface in an axisymmetric stagnation flow field.

$$\vec{W} = -2A(z - z^2)\vec{i}_z, \tag{7}$$

where z is one of the coordinates of the cylindrical coordinate system (ω, φ, z), with the origin of the system being located at the point where the particle would just touch the planar (bubble) surface and the z-axis passes through the centre of the particle (see Fig. 1). In Eq. (7), A is a known parameter independent of z (Nguyen, 1999). In relation to the bubble–particle interaction, z is the distance measured from the planar surface and scaled by dividing by the bubble radius. The negative sign before the right hand side of Eq. (7) indicates that the liquid flows towards the plane surface while the unit vector, $\vec{i}$, is pointed outwards into the liquid.

Using the local coordinate system, the axial and radial components of the fluid velocity, U, are related to the undisturbed flow stream function, ψ_u, by the following expressions:

$$U_\omega(\omega, z) = \frac{1}{\omega}\frac{\partial \psi_u}{\partial z}, \tag{8}$$

$$U_z(\omega, z) = -\frac{1}{\omega}\frac{\partial \psi_u}{\partial \omega}. \tag{9}$$

Solution of Eqs. (8)–(10) and applying the boundary condition:

$$U_z(0, z) = W \tag{10}$$

from Eq. (7), yields:

$$\psi_u = A\left(\omega^2 z - \omega^2 z^2\right). \tag{11}$$

The stream function of the undisturbed fluid ψ_u is now described in terms of the local cylindrical coordinate system (ω, φ, z) and can be used as the boundary conditions for the solution to the flow disturbed by the particle. It will be shown in the Section 2.2.

2.2. Stream function of the disturbed axisymmetric stagnation flow

The stream function, ψ, for the disturbed flow, due to the presence of the particle, can be related to the stream function for the undisturbed flow by the following expression:

$$\psi = \psi_u + \phi, \tag{12}$$

where ϕ is the difference between the two stream functions. For locations far from the particle surface ψ should be equal to ψ_u, which leads to

$$\phi(\infty) = 0. \tag{13}$$

For creeping flow around the particle, then

$$E^2\left(E^2\phi\right) = 0, \tag{14}$$

where the differential operator, E^2, in the reduced cylindrical coordinate system is given by

$$E^2 = \frac{\partial^2}{\partial z^2} - \frac{1}{\omega}\frac{\partial}{\partial \omega} + \frac{\partial^2}{\partial \omega^2}. \tag{15}$$

Eq. (14) needs to be solved for ϕ using the appropriate boundary conditions at the particle and bubble surfaces, in order to find a solution for the disturbed flow stream function, given by Eq. (12).

3. Method and solution for the disturbed flow field

3.1. Transformation of axes

An expression for the disturbed flow stream function can be more readily obtained by applying a transformation of axes from the existing cylindrical system into bispherical coordinates (ξ, φ, η). The relationship between the two coordinate systems is given by Happel and Brenner (1991) as:

$$\omega = \frac{c \sin \eta}{\cosh \xi - \cos \eta} \tag{16}$$

and

$$z = \frac{c \sinh \xi}{\cosh \xi - \cos \eta}, \tag{17}$$

where c is a positive constant, and is given by the expression:

$$c = \kappa \sinh \alpha, \tag{18}$$

where κ is the particle radius made dimensionless by dividing by the characteristic length. In this case the characteristic length is the bubble radius, which is much larger than the radius of the particle.

3.2. Expression for ϕ

A solution to Eq. (14) is first obtained in an analytical closed form in the bispherical coordinates by Stimson and Jeffery (1926) (Happel and Brenner, 1991). With slight modifications for later convenience, the solution is given by

$$\phi = Ac^3(\cosh \xi - \sigma)^{-3/2} \sum_{m=1}^{\infty} N_m(\xi) C_{m+1}^{-1/2}(\sigma), \tag{19}$$

where

$$\sigma = \cos \eta, \tag{20}$$

and $C_{m+1}^{-1/2}(\sigma)$ is the Gegenbauer polynomial of order $(m+1)$ and degree $-1/2$. In Eq. (19) the function $N_m(\xi)$ is described by

$$N_m(\xi) = a_m \cosh(j_m \xi) + b_m \sinh(j_m \xi) + c_m \cosh(k_m \xi) + d_m \sinh(k_m \xi), \tag{21}$$

where $j_m = m - 1/2$ and $k_m = m + 3/2$, while the integration constants a_m to d_m are able to be determined from the boundary conditions applied at both the particle and bubble surfaces.

3.3. Planar (bubble) surface

At the free surface, $\xi = 0$, and applying conditions of zero normal velocity and zero tangential stress, gives

$$\phi|_{\xi=0} = 0, \tag{22}$$

and

$$\left(\frac{\partial^2 \phi}{\partial \xi^2}\right)_{\xi=0} = 0. \tag{23}$$

Inserting Eq. (19) into Eqs. (22) and (23), leads to

$$a_m = c_m = 0. \tag{24}$$

Expressions for b_m and d_m, can be found by applying the boundary conditions at the particle surface.

3.4. Particle surface

At the particle surface, $\xi = \alpha$, and applying a non-slip boundary condition yields:

$$\phi|_{\zeta=\alpha} = -\psi_{\mathrm{u}}|_{\zeta=\alpha} \tag{25}$$

and

$$\left(\frac{\partial \phi}{\partial \xi}\right)_{\xi=\alpha} = -\left(\frac{\partial \psi_u}{\partial \xi}\right)_{\xi=\alpha}. \tag{26}$$

In Eqs. (25) and (26) the undisturbed stream function, ψ_{u}, for the bispherical coordinate system is obtained by substituting the relationships for ω and z, i.e. Eqs. (16) and (17), into Eq. (11). The resultant expression is:

$$\psi_{\mathrm{u}} = Ac^3(1-\sigma^2)\left[\frac{\sinh \xi}{(\cosh \xi - \sigma)^3} - \frac{c \sinh^2 \xi}{(\cosh \xi - \sigma)^4}\right]. \tag{27}$$

The right-hand side of Eq. (27) can be expressed as a series of the Gegenbauer polynomials using the following relationships derived from (Magnus et al., 1966):

$$\frac{(1-\sigma^2)}{(\cosh \xi - \sigma)^{3/2}} = 2\sqrt{2}\sum_{m=1}^{\infty} m(m+1)\exp(-m\xi - \xi/2)C_{m+1}^{-1/2}(\sigma) \tag{28}$$

$$\frac{(1-\sigma^2)}{(\cosh \xi - \sigma)^{5/2}} = \frac{2\sqrt{2}}{3\sinh \xi}\sum_{m=1}^{\infty} m(m+1)(2m+1)\exp(-m\xi - \xi/2)C_{m+1}^{-1/2}(\sigma). \tag{29}$$

Substituting Eqs. (28) and (29) into Eq. (27), and after manipulation, leads to

$$\psi_{\mathrm{u}} = -Ac^3(\cosh \xi - \sigma)^{-3/2}\sum_{m=1}^{\infty} U_m(\xi)C_{m+1}^{-1/2}(\sigma), \tag{30}$$

where

$$U_m(\xi) = -2\sqrt{2}m(m+1)\mathrm{e}^{-\frac{2m+1}{2}\xi}\left[1 - \frac{c(2m+1)}{3}\right]\sinh\xi. \tag{31}$$

Solving Eqs. (25) and (26), simultaneously, using Eqs. (19), (24) and (30) leads to

$$b_m \sinh(j_m\alpha) + d_m \sinh(k_m\alpha) = U_m(\alpha), \tag{32}$$

$$j_m b_m \cosh(j_m\alpha) + k_m d_m \cosh(k_m\alpha) = \left(\frac{\mathrm{d}U_m(\xi)}{\mathrm{d}\xi}\right)_{\xi=\alpha}. \tag{33}$$

Solving the linear Eqs. (32) and (33), simultaneously, gives

$$b_m = \frac{\left(\mathrm{d}U_m/\mathrm{d}\zeta|_{\zeta=\alpha}\right)(\sinh(j_m\alpha) - \sinh(k_m\alpha))}{k_m\cosh(k_m\alpha)\sinh(j_m\alpha) - j_m\cosh(j_m\alpha)\sinh(k_m\alpha)}, \tag{34}$$

and

$$d_m = \frac{U_m(k_m\cosh(k_m\alpha) - j_m\cosh(j_m\alpha))}{k_m\cosh(k_m\alpha)\sinh(j_m\alpha) - j_m\cosh(j_m\alpha)\sinh(k_m\alpha)}. \tag{35}$$

4. Force exerted by the flow on the particle

For a spherical particle approaching a planar surface in an axisymmetric stagnation flow, the force, F_2 exerted by the liquid on the particle is given by Happel and Brenner (1991). Adapting to the notations of this paper we obtain:

$$\vec{F}_2 = \vec{i}_z(\pi\mu R/\kappa)\int \omega^3 \frac{\partial}{\partial n}\left(\frac{E^2\psi}{\omega^2}\right)\mathrm{d}s, \tag{36}$$

where n and s are the normal and tangential coordinates, respectively, to the particle surface, i.e.

$$\frac{\partial}{\partial n} = -\frac{\cosh\alpha - \sigma}{c}\frac{\partial}{\partial\omega} \tag{37}$$

and

$$\mathrm{d}s = -\frac{c}{\cosh\alpha - \sigma}\mathrm{d}\eta. \tag{38}$$

The substitution of Eq. (12) into Eq. (36) yields

$$F_2 = (\pi\mu R/\kappa)\int \omega^3 \frac{\partial}{\partial n}\left(\frac{E^2\phi}{\omega^2}\right)\mathrm{d}s, \tag{39}$$

which can be integrated over the particle surface ($\xi = \alpha$ and $0 \leqslant \eta \leqslant \pi$) to obtain an expression for the overall force acting on the particle, i.e.

$$F_2 = Ac^3\frac{2\sqrt{\pi}\mu R/\kappa}{c}\sum_{m=1}^{\infty}(a_m + b_m + c_m + d_m), \tag{40}$$

where a_m and c_m are zero, as described by Eq. (24), and b_m and d_m are given by Eqs. (34) and (35), respectively. Given that the sum of b_m and d_m can be expressed as,

$$b_m + d_m = -\frac{4\sqrt{2}m(m+1/2)(m+1)\sinh^2\alpha}{\sinh(2m\alpha+\alpha) - (m+1/2)\sinh(2\alpha)}\left[1 - \frac{c(2m+1)}{3}\right], \tag{41}$$

Eq. (40) can be written as:

$$F_2 = -12\pi\mu AR\left(hf_2' - h^2 f_2''\right), \tag{42}$$

where h is the distance of the particle centre measured from the bubble surface and scaled by dividing by the bubble radius. In terms of α and κ, it can be shown that

$$h = \kappa\cosh\alpha. \tag{43}$$

In Eq. (42) f_2' and f_2'' are equal to

$$f_2' = \frac{4}{3}\frac{\sinh^4\alpha}{\cosh\alpha}\sum_{m=1}^{\infty}\frac{m(m+1/2)(m+1)}{\sinh(2m\alpha+\alpha) - (m+1/2)\sinh(2\alpha)} \tag{44}$$

$$f_2'' = \frac{4}{9}\frac{\sinh^5\alpha}{\cosh^2\alpha}\sum_{m=1}^{\infty}\frac{m(m+1/2)^2(m+1)}{\sinh(2m\alpha+\alpha) - (m+1/2)\sinh(2\alpha)}. \tag{45}$$

Eq. (42) can be used to calculate the flow force acting on a particle approaching a mobile planar (bubble) surface in a fluid flow which has a stream function described by the characteristic parameter, A, defined in Eq. (7). It can be equated to the corrected Stokes' drag force equation, given by Eq. (6) to obtain an expression for the correction factor, f_2, associated with the motion of the fluid. This is described in the Section 5.

5. Discussion

The velocity of the undisturbed flow field is described by Eq. (7). Inserting $z = h$ into this equation and then the obtained result into Eq. (6) yields

$$F_2 = -12\pi\mu RA\left(h - h^2\right)f_2. \tag{46}$$

Equating Eqs. (42) and (46) gives

$$f_2 = \frac{f_2' - hf_2''}{1-h}. \tag{47}$$

It is mentioned that h in the above-described equation can be described in terms of the separation distance H between the particle surface and the interface, the particle radius R and the bubble radius R_b by

$$h = \frac{H+R}{R_b}. \tag{48}$$

As indicated by Eqs. (44) and (45), both f_2' and f_2'' depend only on α which is a function, as can be seen from Eq. (4), of the separation distance, H, between the particle and bubble surfaces. The

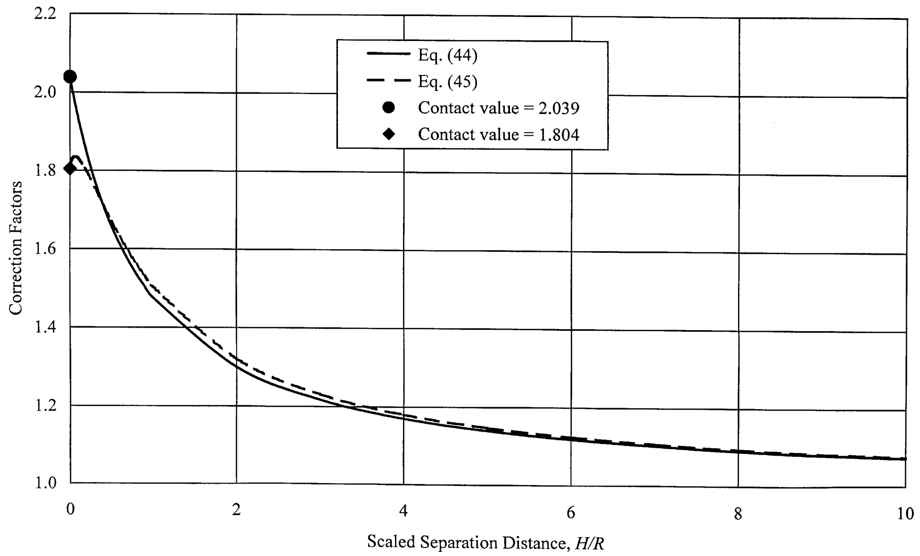

Fig. 2. Correction factors f_2' and f_2'' as a function the ratio of the inter-surface separation distance to the particle radius, H/R.

results are plotted in Fig. 2. It can be seen that for large separation distances, both f_2' and f_2'' approach unity. This result is consistent with the modified Stokes equation described by Eq. (6) since it can be shown that the correction factor f_2, defined by Eq. (47), approaches unity at large separation distances.

For both series given by Eqs. (44) and (45) a singularity exists when the separation distance is zero. To overcome this problem the tangent-sphere coordinate system can be used to determine the contact values of f_2' and f_2'', giving numerical values of 2.039 and 1.084, respectively. These results are also shown in Fig. 2. It can be seen that the predicted values for f_2' and f_2'' are consistent over the entire range of $0 \leqslant H/R \leqslant \infty$, highlighting the significance of the short range hydrodynamic interaction once the particle approaches within a distance of a few diameters from the bubble surface.

As described by Eqs. (44) and (45), the expansion for the drag force on the particle involves the infinite series of transcendental functions. For the bubble–particle interaction modelling exercises, these series are rather complicated and usually require the numerical computational procedures to determine the resistance coefficients as a function of the separation distance. The next task of this paper is to develop simple approximations for f_2' and f_2'' as a function of the ratio of the inter-surface separation distance to the particle radius, H/R. As can seen from Fig. 2, these functions approach the finite values at $H/R = 0$, and unity when H/R is very large. Based on these asymptotes, simple approximations for f_2' and f_2'' are expected to have the functional dependence of the form $(a + H/R)/(b + H/R)$. The numerical constants a and b can be obtained by the best fit to the exact numerical data of Eqs. (44) and (45). The best fit was solved by the non-linear least squares procedure. The resultant approximate expressions for f_2' and f_2'' as a function of H/R are described by Eqs. (49) and (50).

$$f_2' = \frac{1.707 + H/R}{0.836 + H/R}, \tag{49}$$

$$f_2'' = \frac{2.656 + H/R}{1.440 + H/R}. \tag{50}$$

These equations result in relative errors, when compared with the numerical results, of less than 3.5% and represent the global rational approximations for resistance coefficients for the whole range of the separation distance.

For practical application, the result of Eq. (3) for the resistance coefficient f_1 can similarly be replaced by a simple approximation. When the surfaces are far apart ($H \to \infty$), f_1 described by Eq. (3) approaches 1. When $H \to 0$, Eq. (2) provide the correct limit predicted by the lubrication theory, namely, $f = R/(4H)$ for a solid sphere approaching a non-deformable planar slip interface. Based on these asymptotes, the following simple approximate expression was found to be able to describe the resistance coefficient for the whole range of the inter-surface separation distance:

$$f_1 = [1 + (R/4H)^p]^{1/p} \tag{51}$$

The exponent p was found by the best fit to the exact data of Eq. (3) using the non-linear least squares procedure and yields $p = 0.719$. This simplified expression is within 4% relative error compared to the exact data.

The bubble–particle attachment interaction may cause the deformation of the bubble surface. This is the main weakness of the assumption made in this paper that the gas–liquid interface is locally planar in comparison to the curvature of the particle surface. Fortunately, the deformation of the gas–liquid interface is a function of the particle kinetic energy and is negligibly small (Schulze, 1989) for the particle size of about 50–200 μm and the bubble size of about 1–2 mm which are typically encountered in the flotation separation process. As a result, the outcomes of this paper can be applied to the bubble–particle interaction found in the normal flotation processes. For the coarse particle flotation, the theory presented in this paper requires a modification to consider the local deformation of the gas–liquid interface as the curvature of the local gas–liquid interface is compatible to the curvature of the particle surface and will influence the hydrodynamic interaction at short separation distances (Kim and Karrila, 1991).

6. Conclusion

In this paper a model was developed to describe the hydrodynamic force acting on a fine solid sphere as it approaches an air bubble with a mobile surface in an axisymmetric stagnation flow. The model incorporated a bispherical coordinate system to derive a stream function for the disturbed flow. The stream function was then used to calculate the hydrodynamic flow force acting on the particle as a function of the separation distance between the particle and bubble surfaces. The force equation was related to the modified Stokes equation to obtain an exact numerical solution for the correction factor, f_2. Predictions obtained in this paper involve terms of the scaled separation distance of orders up to 2, and, therefore, are expected to provide sufficiently accurate short-range hydrodynamic force of the bubble–particle interaction. Finally, simplified analytical expressions are presented, which are in good agreement with the exact numerical result, and can be readily applied to more general applications involving the bubble–particle interaction in multiphase flow processes.

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